RIGA TECHNICAL UNIVERSITY 1862

Power and Electrical Engineering

doi: 10.7250/pee.2017.007
2017, vol, 34, pp. 37–42
2017/34

Grid Study on the Basis of Reciprocal Power Losses Calculation

Josifs Survilo*
Riga Technical University, Latvia

***Abstract* – It was found that the losses of some supplier-consumer pairs in a grid (reciprocal losses) can be negative. Upon exploring this issue, it turned out that this phenomenon is not rare and is a result of a certain distribution of the power of electricity suppliers between consumers. Negative reciprocal losses indicate that the overall grid losses are increased. They can be eliminated by redistributing the suppliers' power. Computing the reciprocal losses, additional properties of a power system can be defined, which facilitates the determination of its more economical operation mode.**

***Keywords* – Node voltage, power grid, power loss, reciprocal current, vector-matrix.**

I. Introduction

The price of electricity depends on many factors, to recall the main of these: the suppliers' or power plants' price, the cost of management, administration and maintenance of the infrastructure, support of renewable energy [1] as well as power losses. The choice of suppliers primarily depends on the price and amount of electricity bought [2] whereas power losses have a secondary role.

Once the suppliers have been selected, optimization of the power system mode is a permanent effort of those dealing with electricity. Now, attention is paid to power losses: computer programs are dedicated to optimal power flow that reduces losses [3], various scientific literature has long been devoted to this subject, researching issues related to losses of electricity transfer, grid development, construction and operation [4], [5]. Reference [6] directly links the economy with losses.

Of particular interest is allocation of power losses, in other words, reciprocal losses (RL), which makes it possible to apply more equitable compensation for electricity and to set fairer overheads.

This issue is receiving a lot of attention. Here are some examples of the proposed methods of load allocation and their shortcomings.

In [7], the supplier-consumer losses are not found directly but in a roundabout manner, i.e. by summation. Power allocation exclusively for radial grids is considered in [8]. A comparison of different loss allocation algorithms is made in [9]. In the literature, one can see cumbersome formulas that depend on grid configuration.

The method of loss allocation proposed in [10]–[13] is more accurate and is uniform for all grid configurations. It relies on the reciprocal current (RC), which is needed to transport a portion of some supplier power (reciprocal power – RP) to certain a consumer. Accompanying RL are defined; besides, sometimes these losses are negative.

The goal of the paper is to study how often the negative RL appear and what is the reason of their occurrence; moreover, how the proposed method of load allocation can be used for deeper characterization of the grid.

Section 2 provides clarification of the formula for partial loss determination. In Section 3, a concise algorithm for computing RL is described and the cause of negative RL is clarified. In Section 4, partial losses and voltage drops in grid elements are considered. In Section 5, the feature to minimize the power losses is considered. Quick determination of voltages at grid nodes is elaborated in Section 6. Conclusions are stated in Section 7.

Voltage, current and power are given in volts, amperes and watts, impedance – in ohms and they are complex quantities unless otherwise shown.

II. Formula for Partial Power Losses

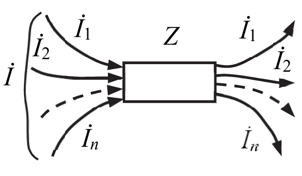

Fig. 1. Impedance Z loaded with a number of currents.

If several currents flow through some impedance (Fig. 1), then the power loss in this impedance caused by some of these currents, e. g. current I_2, is here called partial loss. Based on publications [10]–[13] and Fig. 5, such loss is defined by the following formula:

$$\Delta s_2 = k\frac{Z}{4}\left(\left|\dot I + \frac{\dot I_2}{k}\right|^2 - \left|\dot I - \frac{\dot I_2}{k}\right|\right) = \frac{1}{\kappa}\frac{Z}{4}\left(\left|\dot I + \kappa\dot I_2\right|^2 - \left|\dot I - \kappa\dot I_2\right|^2\right). \quad (1)$$

For convenience in further consideration, substitution $k = 1/\kappa$ is made. In [13] $k = 100$, then

$$\Delta s_2 = 25Z\left(\left|\dot I + 0.01\dot I_2\right|^2 - \left|\dot I - 0.01\dot I_2\right|^2\right).$$

To substantiate a simpler formula, complex numbers are presented as follows:

$$\dot I = I(\cos\alpha + j\sin\alpha);\quad \dot I_2 = I_2(\cos\beta + j\sin\beta). \quad (2)$$

Inserting (2) into expression $c = \left|\dot I + \kappa\dot I_2\right|^2 - \left|\dot I - \kappa\dot I_2\right|^2$, we obtain the following:

---

* Corresponding author.
E-mail address: survilo@eef.rtu.lv

$$c = 4\kappa I I_2[\cos(\alpha-\beta)] = 4\kappa I I_2(\cos\alpha\cos\beta + \sin\alpha\sin\beta). \quad (3)$$

Inserting (3) into (1), we obtain:

$$\Delta s_2 = Z I I_2(\cos\alpha\cos\beta + \sin\alpha\sin\beta). \quad (4)$$

Whatever number we have assigned to k or κ in (1), we still get (4). When we insert $k = 1$ into (1), we have

$$\Delta s_2 = 0.25Z(|\dot{I} + \dot{I}_2|^2 - |\dot{I} - \dot{I}_2|^2). \quad (5)$$

Reduction k times of partial current (it was reciprocal current) was necessary at the very beginning [10], searching for RP.

The sum of partial power losses, which are computed by formulas based on expression (1), is equal to the power loss caused by the summary current (which corresponds to the essence of electrical engineering):

$$\Delta S = ZI^2 = \Delta s_1 + \Delta s_2 + \ldots + \Delta s_n. \quad (6)$$

The formula of other type of partial losses is the sum of losses equaled to summary current loss.

III. Reciprocal Losses

The research is being conducted on a 330 kV radial grid (Fig. 2) and on a closed grid (Fig. 3).

For simplification purposes, permittance is assumed zero. In [12], it is shown how power line admittances can be dealt with.

Twenty cases with adequate node power in accordance with Table I are considered to facilitate and illustrate the task.

For each case, the RL Δs_{jtk} has to be computed first, where Δs_{jtk} is the power spent for transmitting a portion of electricity (the above-mentioned RP) from a specified supplier 'j', which reaches (in the index, the letter 't' stands for 'to') a certain consumer 'k'. The principal quantity for computing RL and other quantities including RP, is reciprocal current RC [13], the current which delivers RP from supplier 'j' to consumer 'k'.

In this paper, the algorithm of for the determination of RL is called the double algorithm (DA) since it involves the use of two computer programs: any power flow program (here, *Power World*) and *Matlab*. The DA is described just further.

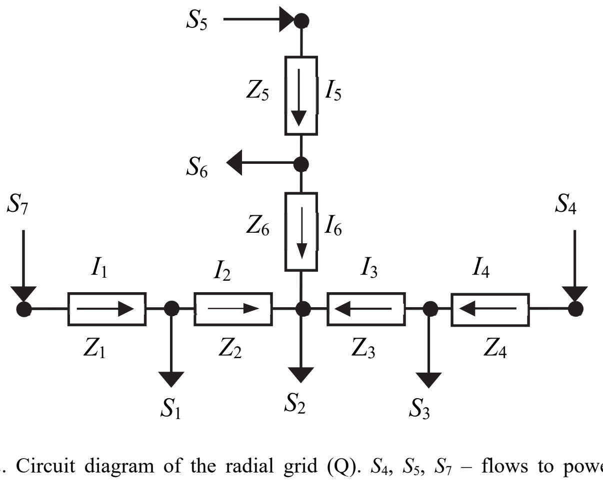

Fig. 2. Circuit diagram of the radial grid (Q). S_4, S_5, S_7 – flows to power-supplying points; S_1, S_2, S_3, S_6 – flows to consumers; I – branch currents; Z – transmission line impedances: $Z_1 = (0.785 + 3.2i)$ Ω, $Z_2 = (5.58 + 28.8i)$ Ω, $Z_3 = (7.425 + 24i)$ Ω, $Z_4 = (1.782 + 5.76i)$ Ω, $Z_5 = (3.465 + 11.2i)$ Ω, $Z_6 = (3.366 + 10.88i)$ Ω.

TABLE I

Load of Power Grid Nodes

Case	S_1	S_2	S_3	S_4	S_5	S_6
	MW, Mvar i					
Qa				100 + 14i	50 + 5i	
Qc				75 + 8.5i	75 + 8.5i	
Qc1				60 + 6.8i	90 + 10.2i	
Qc2				67.5 + 7.65i	82.5 + 9.35i	
Q2i		208 + 39i				
Q2i3		256 + 48i				
Q2r		100 + 100i				
Q4i3				160 + 50i		
Q4r				70 + 50i		
Fa				100 + 14i	40	
Fc				56 + 5.6i	84 + 8.4i	
Fc1				42 + 4.2i	58 + 5.8i	
Fc2				25.8 + 2.58i	34.2 + 3.42i	
Fc3				18.48 + 1,848i	23.52 + 2.35i	
Fc4				12.43 + 1.243i	15.57 + 1.57i	
Fc5				15.68 + 1.568i	19.32 + 1.93i	
F2d		50 + 20i				
F2r		40 + 100i				
F4w6				0		
F4w4				50 + 7i		

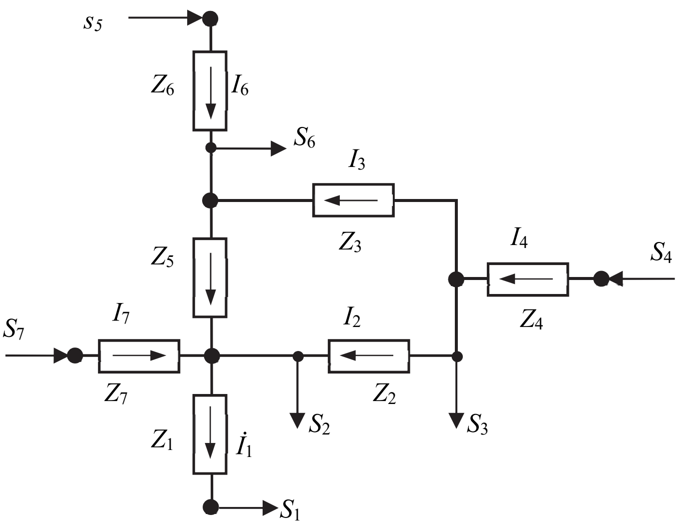

Fig. 3. Circuit diagram of the closed grid (F)
S_4, S_5, S_7 – flows to power supplying points; S_1, S_2, S_3, S_6 – flows to consumers; I – branch currents; Z – transmission line impedances:
$Z_1 = (11.22 + 70.4i)$ Ω, $Z_2 = 6.1 + 32i)$ Ω, $Z_3 = (6.48 + 25.6i)$ Ω, $Z_4 = 11.125 + 80i)$ Ω, $Z_5 = (1.02 + 6.4i)$ Ω, $Z_6 = (10.2 + 64i)$ Ω, $Z_7 = (0.81 + 3.2i)$ Ω.

Briefly, using the circuit diagram shown in Fig. 2, to determine RL, the following should be done: 1) using *Power World*, determine node voltages $U_1 \ldots U_7$ and their angles $\alpha_1 \ldots \alpha_7$ relative to the slack bus 7 (the indices of the voltages and nodes coincide with those of node power S); 2) determine node currents $J_1 \ldots J_7$ by means of *Matlab*; 3) determine branch admittances for the studied grid, $Y_1 \ldots Y_6$; 4) determine consumer admittances Y_{c1}, Y_{c2}, Y_{c3}, Y_{c6} by means of *Matlab*; 5) build conductance matrix $\boldsymbol{Y}$ and its inverse matrix $\boldsymbol{Z}_v$; 6) build matrix $\boldsymbol{A}$ and its inverse matrix $\boldsymbol{B}$; 7) build matrix $\boldsymbol{Z}$; 8) build node current vector-matrix $\boldsymbol{J}_{v4}$, $\boldsymbol{J}_{v5}$, $\boldsymbol{J}_{v7}$ to determine the node voltages; 9) determine supplier-consumer phase node voltages $U_{v4t1}...U_{v7t6}$ to compute RC; 10) perform calculations according to formula batch (7):

$$I_{4t1} = Y_{c1}U_{4t1}, ..., I_{7t6} = Y_{c6}U_{7t6}; J_{4i4t1} = J_4 + I_{4t1}; J_{1i4t1} = J_1 + I_{4t1}; \boldsymbol{J}_{i4t1}=[-J_{1i4t1}, \ -J_2, \ -J_3, \ J_{4i4t1}, \ J_5, \ -J_6]; \ \boldsymbol{I}_{i4t1} = \boldsymbol{BJ}_{i4t1}; \Delta s_{i4t1}=\boldsymbol{I}_{i4t1}'\boldsymbol{ZI}_{i4t1}; \ J_{4d4t1}=J_4 - I_{4t1}; \ J_{1d4t1}=J_1 - I_{4t1}; \ \boldsymbol{J}_{d4t1} = ...; \boldsymbol{I}_{d4t1} = ...; \ \Delta s_{d4t1} = ...; \ \Delta s_{4t1} = 0.25(\Delta s_{i4t1} - \Delta s_{d4t1}); ...; \ \Delta s_{7t6} = ...; s\Delta s=\Delta s_{4t1} + ... + \Delta s_{7t6}; \ \boldsymbol{J} = [-J_1, -J_2, -J_3, J_4, J_5, -J_6]; \boldsymbol{I} = \boldsymbol{BJ}; \Delta s = \boldsymbol{I'ZI}; \boldsymbol{J}_v = [0, 0, 0, J_4, J_5, 0, J_7]; \boldsymbol{U}_v = \boldsymbol{Z}_v\boldsymbol{J}_v, \quad (7)$$

where I_{4t1}; ...; I_{7t6} are the RCs for all the pairs between suppliers and consumers; J_{4i4t1} – node current of supplier 4 incremented by RC I_{4t1}; J_{1i4t1} – the same for consumer node 1; $\boldsymbol{J}_{i4t1}$ – incremented node 4 current vector; $\boldsymbol{I}_{i4t1}$ –branch current vector (the currents in all branches) of node 4 incremented by I_{4t1}; $\boldsymbol{I}_{i4t1}'$ – conjugate transposed current vector $\boldsymbol{I}_{i4t1}$; Δs_{i4t1} –power loss for one phase in the grid, incremented by RC I_{4t1}; J_{4d4t1} – node current of supplier 4 decremented by RC I_{4t1}; J_{1d4t1} – the same for consumer node 1; ...; Δs_{4t1} – RL for one phase of supplier 4 to consumer 1; $s\Delta s$ – sum of RC for one phase of all supplier-consumer pairs; $\boldsymbol{I}$ – branch current vector; Δs – total losses of the grid; $\boldsymbol{J}$ – supplier node current vector; $\boldsymbol{U}_v$ – node phase voltage vector.

The size of the paper does not allow a more exhaustive description of the algorithm. Details are to be found in [10]–[13]. However, the best idea about DA can be got by acquainting oneself with the *Matlab* file.

To make sure that no errors have not found their way into the DA calculations, the sum of the RLs, $s\Delta s$, should be compared with total grid loss Δs; at least four to five digits should match.

Specific data calculated for case Qa are given in Table II and losses – in Table III.

Total losses Δs and sum of RL, $s\Delta s$, are as follows in Qa:

Δs = 337 130 + 1259 600i; $s\Delta s$ = 337 140 + 1259 600i.

TABLE II

NODE CURRENTS J, VOLTAGES U_v AND BRANCH CURRENTS I FOR CASE Qa

No	$J_{1...7}$	$U_{v1...7}$	$I_{1...6}$
1	70.09 − 0.2i	190 234.4 − 521.7i	176.73 − 48.58i
2	282.29 − 58.1i	188 245.9 − 3322.9i	106.64 − 48.38i
3	35.19 − 17.6i	189 435 + 3.1i	140.47 − 6.09i
4	175.66 − 23.69i	189 884.5 + 972.7i	175.66 − 23.69i
5	88.16 − 9.77i	188 818.6 − 1998.7i	88.16 − 9.77i
6	52.98 − 6.14i	188 403.7 − 2952.3i	35.18 − 3.63i
7	176.73 − 48.58i	190 528.6 + 5.7i	

TABLE III

RL ON THE PATH FROM SUPPLIER 4, 5, 7 TO CONSUMER 1, 2, 3, 6 OF CASE Qa

Con-sumer	Supplier 4 $\Delta s_{4t1,2,3,6}$	Supplier 5 $\Delta s_{5t1,2,3,6}$	Supplier 7 $\Delta s_{7t1,2,3,6}$
1	20 934 + 36 622i	−2195 – 22 479i	4086 + 16 658i
2	154 210 + 498 440i	24 360 + 78 740i	93 347 + 463 380i
3	4851 + 15 681i	−4387 – 14 181i	−1868 + 15 691i
6	26 081 + 84 302i	3270 + 10 570i	14 449 + 76 158i

For case Fa, the above-mentioned types of data are given in Table IV and V.

In Fa: Δs = 552 510 + 3581 100i; $s\Delta s$ = 552 510 + 3581 100i.

So, the radial grid Qa has even three types of negative reciprocal losses. To find out the reason of negative RL, let us calculate RL Δs_{5t3} of case Qa, transmitting the electricity from supplier 5 to consumer 3. This is done without applying matrix algebra, using the current values calculated by DA. RC flows through the impedances Z_3, Z_5, Z_6. These currents in the above-mentioned impedances are as follows: I_{3I5t3}= −7.0907 + 3.0637i; I_{5I5t3} = 7.0907 − 3.0637i; I_{6I5t3} = 7.0907 − 3.0637i. Through Z_1, Z_2, Z_4, RC I_{5t3} does not flow. The total current in Z_3, Z_5, Z_6 is as follows: I_3 = 140.47 − 6.09i; I_5 = 88.16 − 9.77i; I_6 = 35.18 – 3.63i. RL Δs_{5t3} is the sum of the partial losses in impedances Z_3, Z_5, Z_6. Observing (5), we have the following: $\Delta s_{3i5t3} = 0.25\ Z_3(|I_3 + I_{3i5t3}|^2 - |I_3 - I_{3i5t3}|^2)$; ...; ... . Their values are: Δs_{3i5t3} = −7534 – 24 353i; Δs_{5i5t3} = 2269 + 7335i; Δs_{6i5t3} = 877 + 2835i, the sum of which is Δs_{5t3} = −4388 – 14 183i. The RL in impedance Z_3 is negative because current I_{3i5t3} flows in the direction opposite to total current I_3.

TABLE IV

NODE CURRENTS J, VOLTAGES U_v AND BRANCH CURRENTS I FOR CASE FA

No	$J_{1...7}$	$U_{v1...7}$	$I_{1...6}$
1	70.38 − 2.03i	189 277.3 − 5461.4i	70.38 − 2.03i
2	280,24 − 53.36i	190 210 − 529.1i	79.69 − 5.13i
3	17.48 − 1.56i	190 860.1 + 1989.7i	75.83 − 3.38i
4	173 − 10.07i	193 589.9 + 15 717.8i	173 − 10.06i
5	52.56 − 5.24i	190 282.3 + 70.3i	93.09 + 3.53i
6	69.81 + 1.67i	190 887.5 + 4555.1i	69.81 + 1.67i
7	177.85 − 53.79i	190 526.2 − 3.5i	177.85 − 53.79i

TABLE V

RL ON THE PATH FROM SUPPLIER 4,5,7 TO CONSUMER 1, 2, 3, 6 FOR CASE FA

con-sumer	supplier 4 $\Delta s_{4t1,2,3,6}$	supplier 5 $\Delta s_{5t1,2,3,6}$	supplier 7 $\Delta s_{7t1,2,3,6}$
1	92 800 + 609 230i	18 155 + 114 630i	28 654 + 169 110i
2	286 120 + 1905 100i	37 304 + 237 040i	18 750 + 74 074i
3	13 868 + 99 726i	758 + 7274i	−868 – 14 510i
6	51 199 + 340 240i	6186 + 38 814i	1585 + 442i

The precondition for negative RL lies in opposite directions of the total current in some impedance and its reciprocal current. When the sum of losses is negative, we have negative RL. Negative power loss in any impedance causes an increase of the total losses in a grid.

IV. Partial Quantities in the Elements of a Grid

Partial power losses caused by a node current or a reciprocal current can be computed in any line or transformer. This can also be done for voltage drop. It is shown using case Qa.

The currents $I_{1...6}$ in lines 1...6 (the line or branch numbers coincide with the impedance indices in Fig. 2) caused by the currents of a certain node, be it a supplier or a consumer (here, the currents caused by node 4):

$$I_{1...6,\,J4\mathrm{o}} = \boldsymbol{BJ}_{4\mathrm{o}};\ \boldsymbol{J}_{4\mathrm{o}} = [-I_{4\mathrm{t}1}, -I_{4\mathrm{t}2}, -I_{4\mathrm{t}3}, J_4, 0, -I_{4\mathrm{t}6}]. \quad (8)$$

These currents are as follows: $I_{1J4\mathrm{o}} = 0$; $I_{2J4\mathrm{o}} = -0.2735 - 0.0067i$; $I_{3J4\mathrm{o}} = 1.6096 - 0.1781i$; $I_{4J4\mathrm{o}} = 1.7566 - 0.2369i$; $I_{5J4\mathrm{o}} = 0$; $I_{6J4\mathrm{o}} = -0.2099 + 0.0142i$.

The power loss of one phase in a branch, for example, branch 2, caused by any node (in this case, node 4), is as follows:

$$\Delta s_{2J4\mathrm{o}} = 0.25Z_2(|I_2 + I_{2J4\mathrm{o}}|^2 - |I_2 - I_{2J4\mathrm{o}}|^2), \quad (9)$$

where I_2 is the current in branch 2 computed by formula $\boldsymbol{I} = \boldsymbol{BJ}$ in batch (7).

The calculated losses in branches 2, ..., 6, caused by supplier 4, are as follows: $\Delta s_{2J4\mathrm{o}} = -16\,094 - 83\,064i$; $\Delta s_{3J4\mathrm{o}} = 168\,680 + 543\,240i$; $\Delta s_{4J4\mathrm{o}} = 55\,986 + 180\,970i$; $\Delta s_{6J4\mathrm{o}} = -2502.9 - 8090.2i$.

The losses in a particular line (any of lines 1...6) as a result of some RCs are calculated in a similar way; let us consider the currents in lines 1...6 caused by RC $I_{4\mathrm{t}1}$:

$$I_{1...6\mathrm{i}4\mathrm{t}1} = \boldsymbol{BJ}_{\mathrm{i}4\mathrm{t}1};\ \boldsymbol{J}_{\mathrm{i}4\mathrm{t}1} = [-I_{4\mathrm{t}1}, 0, 0, I_{4\mathrm{t}1}, 0, 0]. \quad (10)$$

To find the loss in branch 2 $\Delta s_{2\mathrm{i}4\mathrm{t}1}$ caused by RC $I_{4\mathrm{t}1}$, current $I_{2J4\mathrm{o}}$ in expression (9) must be substituted by $I_{2\mathrm{i}4\mathrm{t}1}$ defined by (10), which flows in the branch.

The voltage drop across some branch is a linear function of current and can be easily found if the current is known. E.g. voltage drop $\Delta u_{2\mathrm{i}4\mathrm{t}1}$ across branch 2 from RC $I_{4\mathrm{t}1}$ is as follows:

$$\Delta u_{2\mathrm{i}4\mathrm{t}1} = Z_2 I_{2\mathrm{i}4\mathrm{i}1} \quad (11)$$

and equals $-133 - 791i$; this current is negative because in Fig. 2 current $I_{2\mathrm{i}4\mathrm{t}1}$ flows in a direction opposite to the one in impedance Z_2. The voltage drop of the total current in branch 2 is $1988.4 + 2801.3i$.

These secondary quantities show the influence of the supplier or consumer on the mode of the grid and indicate severe overloads of grid elements and their origin.

V. Optimum Use of Power Suppliers

Among other factors, losses depend on the distribution of power required by the consumers between the suppliers [14]. It can be said that if the price is equal for all the suppliers, then, the farther the supplier is located, the less electricity has to be bought to reduce the losses in the grid. It was found that an optimal load distribution occurs when the active components U_d of the distinctive voltages mentioned here on the way from the supplier to the generalized load (see Fig. 4) are equal for all the suppliers.

The distinctive voltages of suppliers 4; 5; 7 are defined as follows:

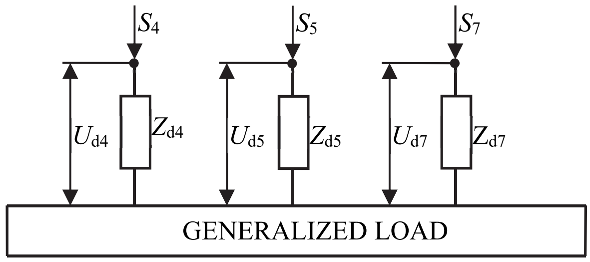

Fig. 4. Representation of the grid with the view of its optimization.

$$U_{\mathrm{d}4} = \Delta s_4 / |J_4|;\ U_{\mathrm{d}5} = \Delta s_5 / |J_5|;\ U_{\mathrm{d}7} = \Delta s_7 / |J_7|, \quad (12)$$

where Δs_4; Δs_5; ...; Δs_7 are the power losses due to the respective suppliers:

$$\Delta s_4 = \Delta s_{4\mathrm{t}1} + \Delta s_{4\mathrm{t}2} + \Delta s_{4\mathrm{t}3} + \Delta s_{4\mathrm{t}6};\ \Delta s_5 = \Delta s_{5\mathrm{t}1}...;$$
$$\Delta s_7 = \Delta s_{7\mathrm{t}1}\ldots. \quad (13)$$

Summary loss Δs is shown in Table VI with appropriate (different) power distributions (see Table I) between the suppliers, which shows that power losses are minimal when the active components of distinctive voltages are equal or close to each other.

It can be seen that the closer the active components of the distinctive voltages each other, the less the active losses in the grid.

TABLE VI

Supplier Distinctive Voltages U_D and Corresponding Active Loss ΔP

Case	$U_{\mathrm{d}4}$	$U_{\mathrm{d}5}$	$U_{\mathrm{d}7}$	Δp
Qa	1163 + 3583i	237 + 594i	600 + 3120i	337 130
Qc	769 + 2313i	550 + 1605i	614 + 3172i	288 265
Qc1	531 + 1545i	736 + 2207i	618 + 3187i	287 684
Qc2	650 + 1929i	643 + 1906i	616 + 3179i	285 161
Fa	2562 + 17 04 8i	894 + 5696i	248 + 1233i	552 510
Fc	1500 + 10 03 6i	1717 + 10 799i	251 + 1243i	443 389
Fc1	1140 + 7636i	1217 + 7658i	317 + 1526i	286 719
Fc2	724 + 4868i	755 + 4751i	384 + 1812i	201 245
Fc3	536 + 3611i	546 + 3438i	415 + 1942i	185 956
Fc4	380 + 2573i	390 + 2457i	439 + 2044i	184 200
Fc5	463 + 3128i	464 + 2921i	427 + 1993i	183 942

Negative RLs quickly disappear when the distinctive voltages approach is used. In case Qa, we have three negative

RL values (Table III) while in case Qc, there are none of them. In case Fa, there is only one negative RL value (Table V) but it persists in case Fc and Fc1 and disappears only in Fc2. So, the total loss Δp is greater when there are negative RLs.

Distinctive impedances

$$Z_{d4} = \Delta s_4/|J_4|^2;\ Z_{d5} = \Delta s_5/|J_5|^2;\ Z_{d7} = \Delta s_7/|J_7|^2 \quad (14)$$

vary relatively little when loads change:
Qa: $Z_{d4} = 6.56 + 20.21i$; $Z_{d5} = 2.68 + 6.69$; $Z_{d7} = 3.37 + 17.02i$;
Qc2: $Z_{d4} = 5.43 + 16.1i$; $Z_{d5} = 4.4 + 13.04i$; $Z_{d7} = 3.35 + 17.3i$.

For a closed grid:
Fa: $Z_{d4} = 14.78+98.37i$; $Z_{d5} = 12.8 + 81.6i$; $Z_{d7} = 1.34 + 6.64i$;
Fc5: $Z_{d4} = 16.81 + 113.51i$; $Z_{d5} = 13.66 + 86.03i$; $Z_{d7} = 1.17 + 5.47i$.

On the basis of this feature, it can be said that the power loss of a supplier is approximately proportional to the square of the supplier's power.

VI. Quick Determination of Node Voltages

When the current of a supplier node changes, the voltage in other nodes changes in the same direction; when change takes place at a consumer node, the change takes place in the opposite direction. The voltage change takes place almost linearly to power change. The sensitivity of any node voltage to power of any other node depends on how far apart electrically they are located.

At first, the most common mode of a power system (the basic mode) should be chosen and the node voltages U_v should be calculated by DA. The second mode (auxiliary) should be calculated by DA as well. Using these two modes, the sensitivity factors should be obtained, which are used for quick calculation of node voltages of other (eventual) mode of a grid. The eventual mode is the mode when the current of some node differs from that of the basic mode.

In Fig. 5 the circuit diagram is shown, explaining the idea.

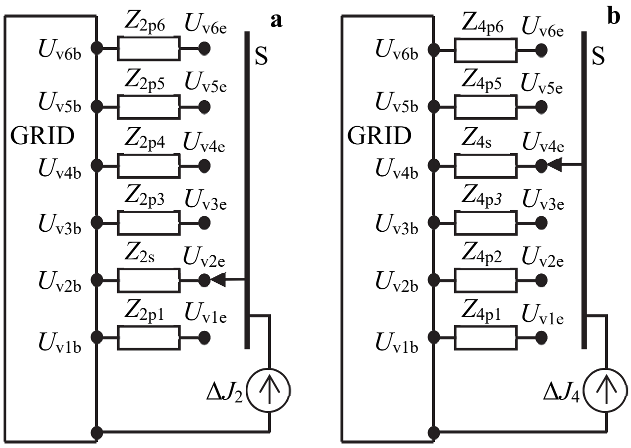

Fig. 5. Updating the voltages of a grid (a) as a result of change of consumer current J_2; (b) change of supplier current J_4.

The power system for this purpose can be considered as a collection of voltage sources [15]. It can be represented by its no-load voltage U_{vib} (i stands for 1...6, b – for 'basic') and by the appropriate impedance. U_{v7}, which is shown in Table II, is not needed since it is slack bus voltage. Impedance Z (putative impedance) for nodes 4 and 2, along with other data, is given in Tables VII and VIII.

TABLE VII

Data of Auxiliary Case Q4I3 for Quick Update (Impact of Node 4)

No	$J_{1\ldots6}$	$U_{v1\ldots6}$	Z_{ipk}
1	69.99 − 0.086i	190 500.4 − 244.3i	$Z_{4p1} = 0.939 + 3.255i$
2	279.45 −530.0i	190 774.8 − 397.8i	$Z_{4p2} = 7.747 + 33.195i$
3	34.78 − 16.29i	194 009.8 + 4947.2i	$Z_{4p3} = 15.439 + 57.338i$
4	276.09 − 76.43i	194 942.1 + 6401.3i	$Z_{4s} = 17.221 + 63.098i$
5	87.15 − 8.29i	191 319.7 + 917.8i	$Z_{4p5} = 7.566 + 33.014i$
6	52.38 − 5.25i	190 924.9 − 29.6i	$Z_{4p6} = 4.467 + 31.727i$

In Table VII, putative impedances are given for the supplier node and are positive.

TABLE VIII

Data of auxiliary case Q2I for quick update (Impact of Node 2)

No	$J_{1\ldots6}$	$U_{v1\ldots6}$	Z_{ipk}
1	70.14 − 0.29i	190 080 − 780i	$Z_{2p1} = -0.924 - 3.257i$
2	368.15 − 81.14i	187 030 − 5910i	$Z_{2s} = -5.667 - 31.652i$
3	35.16 − 18.19i	188 270 − 2590i	$Z_{2p3} = -5.096 - 31.569i$
4	176.39 − 26.24i	188 740 − 1620i	$Z_{2p4} = -4.875 - 31.505i$
5	88.56 − 11.05i	187 620 − 4590i	$Z_{2p5} = -5.466 - 31.648i$
6	53.22 − 6.92i	187 190 − 5540i	$Z_{2p6} = -5.641 - 31.653i$

In Table VIII, they are given for the consumer node and are negative. When the grid mode is basic, the current change $\Delta J = 0$ and the eventual voltage on the outlet $U_{ve} = U_{vb}$; when the current of some node changes (eventual mode), the current change $\Delta J \neq 0$, this current is fed to the outlet, a voltage drop across the putative impedance appears and the eventual voltage U_{ve} is not equal to the basic one. Putative impedance is designated Z_{is} when voltage is considered on that very node i where current ΔJ_i appears; it is designated Z_{ipk} when the voltage U_{vk} on some (other than i) node k of this grid is considered.

Eventual voltage U_{vi} or U_{vk} of node i or k is calculated, imagining that current change ΔJ_i in node i is fed to this very node i through switch S.

To find putative impedances, node voltages U_{v1b}, ..., U_{v6b}, U_{v1a}, ..., U_{v6a} and node currents J_{1b}, ..., J_{6b}; J_{1a}, ..., J_{6a} in the basic and auxiliary mode of the grid should be obtained by DA.

Then impedances Z_{is} and Z_{ipk} are defined as follows:

$$Z_{is} = \frac{U_{via} - U_{vib}}{J_{via} - J_{vib}};\ Z_{ipk} = \frac{U_{vka} - U_{vkb}}{J_{via} - J_{vib}}. \quad (15)$$

Naturally, the value of Z_{is} is greater than the value of any Z_{ipk} since the voltage at this node i is the most sensitive to its current. These impedances show which nodes are more prone to voltage variations.

Eventual U_{vi}; U_{vk} voltages are found by the following expressions:

$$U_{vie} = U_{vib} + Z_{is}(J_{ie} - J_{ib});\ U_{vke} = U_{vkb} + Z_{ipk}(J_{ie} - J_{ib}). \quad (16)$$

This time, J_{ie} is not obtained by DA because we do not use DA or any other program (as it is inconvenient at the moment) but use the quick method, using the known basic voltage U_{vb} and the changed power S_{ie}:

$$J_{ie} = \frac{\hat{S}_{ie}}{3\hat{U}_{vib}}. \quad (17)$$

If there is doubt about the accuracy, the node current J_{ie} can be recalculated by (17) using the voltage U_{vie} obtained by (16):

TABLE IX

THE ACCURACY OF QUICK CALCULATION OF NODE VOLTAGE CHANGES

Case base/auxil./event.	d_{1e} / $d_{1e}^{(1)}$	d_{2e} / $d_{2e}^{(1)}$	d_{3e} / $d_{3e}^{(1)}$	d_{4e} / $d_{4e}^{(1)}$	d_{5e} / $d_{5e}^{(1)}$	d_{6e} / $d_{6e}^{(1)}$
	0.0460	0.0771	0.0765	0.0780	0.0756	0.0757
	0.0577	0.0053	0.0018	0.0021	0.0040	0.0052
	0.0981	0.0188	0.0375	0.0472	0.0226	0.0197
	0.0576	0.0472	0.0759	0.0864	0.0578	0.0496
	0.0337	0.0602	0.0261	0.0215	0.0488	0.0744
	0.0856	0.0112	0.0732	0.0661	0.0960	0.0439
	0.0332	0.0102	0.0193	0.0969	0.0331	0.0115
	0.0299	0.0098	0.0214	0.0995	0.0357	0.0120
	0.0459	0.0461	0.0454	0.0453	0.0467	0.0460
	0.0009	0.0031	0.0021	0.0021	0.0047	0.0071

d_{ne} is the value computed by (17); $d_{ne}^{(1)}$ – the value computed by (18).

$$J_{ie}^{(1)} = \frac{\hat{S}_{ie}}{3\hat{U}_{vie}}. \quad (18)$$

and the node voltages are calculated again by (16).

To check the accuracy of this method, in this paper eventual modes were calculated also by DA and node voltages U_{vieDA}, U_{vkeDA} were obtained. For any node n, accuracy is defined as the difference between voltage U_{vne} obtained by the quick method and voltage U_{vneDA} obtained by DA; this difference is related to the difference between voltage U_{vneDA} and basic mode voltage U_{vnb}:

$$d_n = \frac{|U_{vne} - U_{vneDA}|}{|U_{vneDA} - U_{vnb}|}. \quad (19)$$

The accuracy results based on (17) and (18) are shown in Table IX. In some cases, the recalculated values are more accurate.

VII. CONCLUSION

1. Negative reciprocal losses indicate increased loss in the grid; the more often they appear, the greater is the total power loss in a grid. Negative losses can be removed and grid losses reduced by a different distribution of power between suppliers.
2. Using the DA algorithm, the power loss and the voltage drop can be determined for any branch of the grid and for any current, be this current caused by any node current or by some reciprocal current.
3. In a grid with minimal losses, the active components of distinctive voltages are equal and the distribution of the supplier power is optimal.
4. For any supplier, distinctive impedance can be obtained, which can be used for quick estimation of the power losses caused by the suppliers.
5. Using putative impedances, the grid node voltages can be updated quickly.

REFERENCES

[1] Electricity trading on the open market. [Online]. Available: www.kp.gov.lv

[2] J. Gerhards, A. Mahnitko, and B. Papkovs, *Control, Optimization of Power Grids and the Risks*. Riga: RTU, 2011, pp. 150–154.

[3] Simulator Overview "Power World". [Online]. Available: www.powerwold.com/produkts/simulator

[4] V. Kazancev, *Loss of Electricity in the Networks of Electrical Grids,* Moscow: Energopublication, 1983.

[5] A. Vanags, *Electrical Networks and Grids*, Riga: RTU, 2007, pp. 156–174.

[6] O. Ozdemir, F. D. Munoz, J. L. Ho, and B. F. Hobbs, "Economic Analysis of Transmission Expansion Planning With Price-Responsive Demand and Quadratic Losses by Successive LP," *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 1096–1107, Mar. 2016. https://doi.org/10.1109/tpwrs.2015.2427799

[7] J.-H. Teng, "Power flow and loss allocation for deregulated transmission systems," *International Journal of Electrical Power & Energy Systems*, vol. 27, no. 4, pp. 327–333, May 2005. https://doi.org/10.1016/j.ijepes.2004.12.005

[8] M. Atanasovski and R. Taleski, "Power Summation Method for Loss Allocation in Radial Distribution Networks With DG," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2491–2499, Nov. 2011. https://doi.org/10.1109/tpwrs.2011.2153216

[9] A. J. Conejo, J. M. Arroyo, N. Alguacil, and A. L. Guijarro, "Transmission loss allocation: a comparison of different practical algorithms," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 571–576, Aug. 2002. https://doi.org/10.1109/tpwrs.2002.800894

[10] J. Survilo, "Reciprocal Load-Generator Power Losses in a Grid," Latvian Journal of Physics and Technical Sciences, 2013, No. 6, vol. 50, pp. 48–63.

[11] J. Survilo, "Account of Losses in Electricity Sales," in *Riga Technical University 54th International Scientific Conference: Digest Book and Electronic Proceedings*, 2013, pp. 77–79.

[12] J. Survilo, "Determination of reciprocal current by electricity delivery to more than one consumer," in *2014 55th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON)*, Oct. 2014. https://doi.org/10.1109/rtucon.2014.6998183

[13] J. Survilo, "A method for node prices formation," in *2015 56th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON)*, Oct. 2015. pp. 283–288. https://doi.org/10.1109/rtucon.2015.7343121

[14] K. Mahmoud, N. Yorino, and A. Ahmed, "Optimal Distributed Generation Allocation in Distribution Systems for Loss Minimization," *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 960–969, Mar. 2016. https://doi.org/10.1109/tpwrs.2015.2418333

[15] L. Neiman and K. Demirchian, *Theoretical Foundations of Electrical Engineering*. Leningrad: Energo Publishing House, 1981, p. 128.